Roll Number		

SET C

INDIAN SCHOOL MUSCAT
FINAL TERM EXAMINATION
MATHEMATICS

CLASS: XII　　Sub. Code: 041　　Time Allotted: 3 Hrs

26.11.2018　　Max. Marks: 100

General Instructions:

(i) All questions are compulsory.
(ii) This question paper contains 29 questions.
(iii) Question 1- 4 in Section A are very short-answer type questions carrying 1 mark each.
(iv) Question 5-12 in Section B are short-answer type questions carrying 2 marks each.
(v) Question 13-23 in Section C are long-answer type questions carrying 4 marks each.
(vi) Question 24-29 in Section D are long-answer type questions carrying 6 marks each.

SECTION A

1. Find the value of $\sin^{-1}\left[\sin\left(\frac{6\pi}{5}\right)\right]$. 1

2. Find a vector of magnitude 7 units, and parallel to the resultant of the vectors $\vec{a} = 2\hat{\imath} + 3\hat{\jmath} - \hat{k}$ and $\vec{b} = \hat{\imath} + 2\hat{\jmath} + \hat{k}$. 1

3. Find the angle between the pair of the lines given by $\frac{x+3}{3} = \frac{y-1}{5} = \frac{z+3}{4}$ and $\frac{x+1}{1} = \frac{y-4}{1} = \frac{z-5}{2}$ 1

OR

Find the angle between the line $\frac{x-2}{3} = \frac{y+1}{1} = \frac{3-z}{-2}$ and the plane $3x + 4y + z = 5$.

4. A random variables X has the following probability distribution 1

X	0	1	2	3	4
P(X)	0	k	2k	5k	3k

Find the value of k

SECTION B

5. A balloon which is always remains spherical, has a variable radius. Find the rate at which its volume is increasing w.r.t its radius when the radius is 10cm. 2

OR

Find the approximate value of f(5.001) ,where f(x) = $x^3 - 7x^2 + 15$.

6. Find the values of k ,if the function f(x) = $\begin{cases} kx+2\ , & x \le \pi \\ \cos x\ , & x > \pi \end{cases}$ is continuous $at\ x = \pi$ 2

7. Show that f(x) = $|x-2|$ is not differentiable at x= 2 2

8. Evaluate: $cos\left(\sin^{-1}\frac{3}{5} + \cos^{-1}\frac{4}{5}\right)$ 2

9. If each element of a second order determinant is either zero or one , what is the probability that the value of the determinant is positive? 2

10. Form the differential equation representing the family of curves $y^2 = a(b^2 - x^2)$ where $a\ and\ b$ are arbitrary constants. 2

11. Find a unit vector perpendicular to each of the vector $\vec{a} = \hat{\imath} - 7\hat{\jmath} + 7\hat{k}\ and\ \vec{b} = 3\hat{\imath} - 2\hat{\jmath} + 2\hat{k}$. 2

OR

Given $|\vec{a}| = 13, |\vec{b}| = 5, and\ \vec{a}.\vec{b} = 60\ .find\ |\vec{a}\ x\vec{b}|$.

12. There are two types of fertilizers $F_1$and F_2 . F_1consists of 10% of nitrogen and 6% phosphoric acid and F_2 consists of 5% of nitrogen and 10% phosphoric acid .After testing the soil conditions a farmer finds that she needs at least 14 kg of nitrogen and 14kg of phosphoric acid for her crops, If F_1 costs Rs6/kg and F_2 costs Rs5/Kg. Formulate the problem so that nutrient requirements are met at a minimum cost. 2

OR

An aeroplane can carry a maximum of 200 passengers. A profit of ₹ 500 is made on each executive class ticket out of which 20% will go to the welfare fund of the employees. Similarly a profit of ₹ 400 is made on each economy class ticket out of which 25% will go for the improvement of facilities provided to the economy class passengers. In both the cases, the remaining profit goes to the airliner's fund. The airline reserves at least 20 seats for executive class. However at least four times as many passengers prefer to travel by economy class than by the executive class. Formulate the problem in order to maximize the net profit of the airline.

SECTION C

13. Solve : $\tan^{-1}\left(\frac{x-1}{x-2}\right) + \tan^{-1}\left(\frac{x+1}{x+2}\right)) = \frac{\pi}{4}$ 4

OR

Prove that $2\tan^{-1}\left(\frac{1}{2}\right) + \tan^{-1}\left(\frac{1}{7}\right) = \tan^{-1}\left(\frac{31}{17}\right)$

14. Find the area of the region bounded by the two parabolas $y = x^2 \ and \ y^2 = x$,using integration. 4

15. If $\hat{a}$ and $\hat{b}$ are unit vectors inclined at an angle θ, then prove that
$\sin\frac{\theta}{2} = \frac{1}{2}\left|\hat{a} - \hat{b}\right|$. 4

16. If $x^{16}y^9 = (x^2 + y)^{17}$. Prove that $\frac{dy}{dx} = \frac{2y}{x}$. 4

OR

Verify Rolles Theorem
$f(x) = e^x(\sin x - \cos x)$ on $[\frac{\pi}{4}, \frac{5\pi}{4}]$
4

17. If $x = a\sin 2t\,(1+\cos 2t)$ and $y = b\cos 2t(1-\cos 2t)$, find $\frac{dy}{dx}$ at $t = \frac{\pi}{4}$.

18. Find τ if the vectors $\vec{a} = \hat{\imath} + \hat{3j} + \hat{k}$, $\vec{b} = \hat{2i} - \hat{\jmath} - \hat{k}$ and $\vec{c} = \tau\,\hat{\imath} + 3\hat{k}$ are coplanar. 4

19. Find the shortest distance between the pairs of lines given by
$$\vec{r} = \hat{\imath} + 2\hat{\jmath} + \hat{k} + \hat{\lambda}(\hat{\imath} - \hat{\jmath} + \hat{k}) and\ \vec{r} = 2\hat{\imath} - \hat{\jmath} - \hat{k} + \mu\,(2\hat{\imath} + \hat{\jmath} + 2\hat{k})$$ 4

20. Find the intervals in which the functions given below are strictly decreasing or strictly increasing:- 4
$f(x) = x^4 - 8x^3 + 22x^2 - 24x + 21$

21. Find the points on the curve $9y^2 = x^3$ where the normal to curve makes equal intercepts with the axes. 4

OR

Find the equation of the tangent to the curve $y = \sqrt{5x - 3} - 2$ which is parallel to the line $4x - 2y + 3 = 0$.

22. A Company has two plants to manufacture scooters. Plant 1 manufactures 70% of the scooters and plant II manufactures 30%. At Plant I 80% of the scooters are rated of standard quality and at plant II 90% of scooters are rated of standard quality. A Scooter is chosen at random and is found to be of standard quality. Find the probability that it has come from plant II. 4

23. Find the general solution of the differential equation 4
$$sec^2 x \tan y\, dx + \ sec^2 y \tan x\, dy = 0 \ .$$

SECTION D

24. Show that the altitude of a right circular cone of maximum volume that can be inscribed in a sphere of radius R is $\frac{4R}{3}$. 6

25. Find the area of the region enclosed between the two circles $x^2 + y^2 = 4$ *and* $(x-2)^2 + y^2 = 4$ 6

OR

Using integration find the area of region bounded by the triangle whose vertices are (1,0),(2,2) and (3,1).

26. Solve the differential equation: $(1+x^2)\frac{dy}{dx} + 2xy = \frac{1}{1+x^2}$ $given\ y = 0\ when\ x = 1.$ 6

OR

Solve: $(x^3 + x^2 + x + 1)\ \frac{dy}{dx} = 2x^2 + x; y = 1\ when\ x = 0$

27. Two Cards are drawn Simultaneously (or successively without replacement) from a well shuffled deck of 52 cards. Find the means variance and standard deviation of the number of queens. 6

28. Find the length and the foot of the perpendicular from the point(2,3,7) to the plane 3x – y - z = 7 , also find image point. 6

OR

Find the equation of the plane passing through the points (1,2,-1), (2,0,2) and parallel to the line $\vec{r} = \left(2\hat{\imath} + \hat{\jmath} + 2\hat{k}\right) + \lambda(\hat{\imath} + 2\hat{\jmath} + 2\hat{k})$.

29. A factory manufactures two types of screws. A and B. Each type of screw requires the use of two machines, an automatic and hand operated. It Takes 4minutes on automatic and 6 minutes on hand operated machines to manufacture a package of screws A, while it takes 6 minutes on automatic and 3 minutes on hand operated machines to manufacture a package of screw B. Each machine is available for at the most 4 hours on any day. The Manufacture can sell a package of screws A at a profit of Rs7 and the screws B at a profit of Rs10. Assuming that he can sell all the screws he manufactures, how many packages of each type should the factory owner produce in a day in order to maximize his profit? Determine the maximum profit? 6

End of the Question Paper